Similarity Rule Between Heat Transfer and Pressure Drop of Porous Materials

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Porous materials are used for many engineering components such as the fixed bed for chemical reactors, the regenerator of the Stirling engine, and the pebble bed of the high-temperature gas-cooled nuclear reactor.

In general, the heat-transfer and pressure-drop characteristics of porous materials cannot be easily specified, except for the fixed beds of uniform, simply-shaped particles of spheres and cylindrical beads. For these simple particles, the characteristic length is usually defined by the diameter d or $6V/S$, which characterizes the heat-transfer and/or pressure-drop properties. However, it is difficult to define it for the porous materials of complex matrices such as foamed metal, fixed beds of nonuniform, and irregularly shaped particles. Thus, for porous materials without sufficient knowledge of its fine structures, experiments are carried out to get information on heat-transfer and/or pressure-drop characteristics. The purpose of this work is to predict the heat-transfer characteristics from the pressure drop data, since heat-transfer experiments are much more difficult.

Much research has been carried out on heat-transfer (Dhingra, 1984; Dixon, 1979; Kudra, 1989) or on pressure-drop characteristics of porous materials (Comite, 1989; Burke, 1928; Ergun, 1949). However, studies that deal with both heat-transfer and pressure-drop characteristics simultaneously are rather scarce. Hamaguchi (1983) adopted mean bore diameter as the characteristic length of the porous metal for pressure drop and mean fiber diameter for heat transfer to yield fairly good correlations. If this choice of characteristic lengths is the best and no other alternative exists, no similarity rule is expected since the bore diameter and the fiber diameter are independent of each other. Unfortunately, Hamaguchi's (1983) experiments are not broad enough to determine the possibility of the existence of the similarity rule. Chiou (1966) tried to define the characteristic length from the pressure-drop data to correlate the heat-transfer data. This is very similar to our approach; however, his results do not seem to be good due to the improper choice of the characteristic length, as will be discussed later.

This work shows that a proper characteristic length is determined by defining the Reynolds number Re and the Nusselt number Nu. Then a universal law or a similarity rule $Nu = f(Re)$ applicable to various porous materials is derived.

Experiments

Experiments have been carried out with various porous materials. To measure the heat-transfer coefficient suitable for each of them, two experimental apparatus were provided: one for steady-state experiment (Kondoh, 1987) and the other for transient experiment (Fukuda, 1990). Details of the materials investigated are listed in Table 1. Test pieces of porous metal or wire gauge are used in the steady-state experiments and others in the transient experiments.

From the experiments, the volumetric heat-transfer coefficient α_v and the pressure drop Δp are obtained.

Derivation of the Similarity Rule between Heat-Transfer and Pressure-Drop Characteristics

It is well known (Reynolds, 1990) that the pressure drop in the porous bed is correlated by a quadratic equation:

$$P/H = a\mu_g u_m + b\rho_g u_m^2, \tag{1}$$

where the first term originates from the frictional resistance in the Poiseuille-type viscous flow, while the second term from the kinetic energy losses in completely turbulent flow. In the limit of zero velocity, the first term in Eq. 1 dominates corresponding to the Darcy's law. For the packed bed of sphere beads Ergun's equation (Ergun, 1952) is applicable:

$$\begin{aligned} a &= 150(1-\epsilon)^2/(\epsilon^3 d_p^2), \\ b &= 1.75(1-\epsilon)/(\epsilon^3 d_p). \end{aligned} \tag{2}$$

According to the form of Eq. 1, it is obviously seen that there are two parameters with the dimension of a length which characterize the pressure drop of the porous beds:

$$l_1 = 1/\sqrt{a}, \qquad l_2 = 1/b. \tag{3}$$

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Table 1. Characteristic Dimensions of Tested Porous Pieces

	Foamed Metal		Wire Gauge	
	#2	#4	*A*	*B*
ϵ (−)	0.952	0.965	0.868	0.946
a (mm^{-2})	15.0	76.3	10.9	3.34
b (mm^{-1})	0.250	0.715	0.745	0.308
l_1 (mm)	0.258	0.115	0.303	0.547
l_2 (mm)	4.00	1.40	1.34	3.24
c	0.0645	0.0818	0.226	0.169
Specification	No. of Cell (1/mm)*		Dia. of Wire (mm)	
	0.43 ~ 0.67	1.0 ~ 1.4	0.58	0.45

Particle	Glass Sphere				Glass Cylinder	Stylene Sphere
	ϕ 4.2 (mm)		ϕ 2.5 (mm)		ϕ 3×5 (mm)	ϕ 1.4 (mm)
ϵ (−)	0.36	0.42	0.38	0.41	0.33	0.40
a (mm^{-2})	92.4	55.6	190	157	135	652
b (mm^{-1})	5.49	3.36	5.78	4.92	7.11	12.5
l_1 (mm)	0.104	0.134	0.0724	0.0796	0.0852	0.0392
l_2 (mm)	0.182	0.298	0.173	0.203	0.141	0.0800
c	0.571	0.451	0.419	0.393	0.612	0.490

* From the data of Cermet, Sumitomo Denko Co.

By drawing a curve $\Delta p/H\mu_g u_m$ vs. $\rho_g u_m/\mu_g$ we get a from the intercept at zero velocity and b from the slope. The combination method of these two characteristic lengths to derive governing parameters (two at the most) is arbitrary and a typical set of parameters is either l_1 or l_2 and the ratio of these two are:

$$c = l_1/l_2. \tag{4}$$

As far as the sphere beads are concerned, Ergun's equation as found to be valid, while it was not for other materials tested. Therefore, in the general case of porous material with complicated structures, the characteristic length other than d_p should be properly chosen. Thus, we are going to look for a similarity rule using the parameters: l_i ($i = 1$ or 2) as the characteristic length and a dimensionless parameter c.

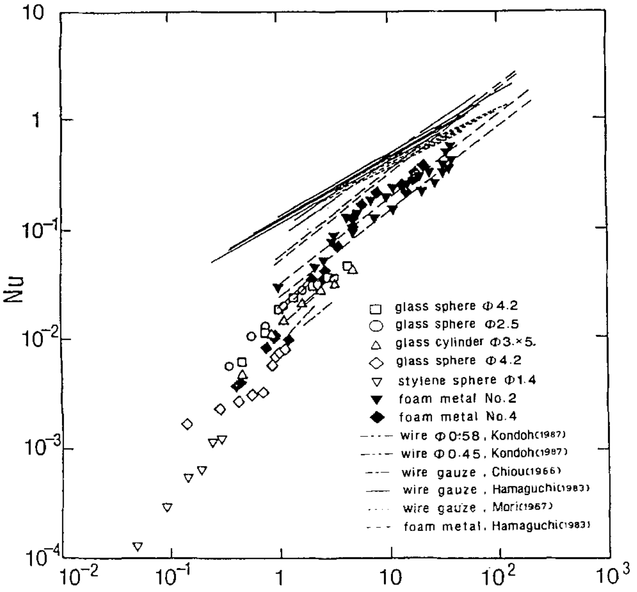

Figure 1. *Nu* vs. *Re* with l_1 chosen as the characteristic length.

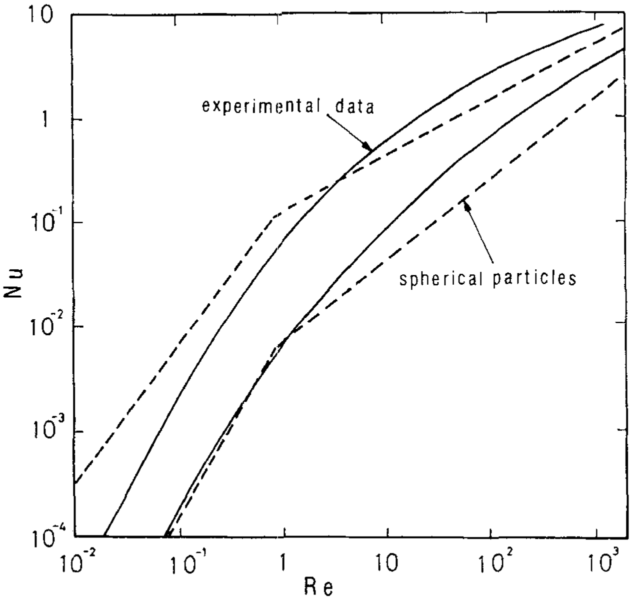

Figure 2. Rough comparison between experimental results and the data for sphere beads bed by other researchers with l_1 chosen as the characteristic length.

Experimental data, together with other available data, are plotted in Figure 1, where l_1 is chosen as the characteristic length for *Re* and *Nu* defined in Eqs. 5 and 6:

$$Nu = \alpha_v l_1^2/\lambda_g, \tag{5}$$

$$Re = u_m l_1/\nu_g. \tag{6}$$

This choice is arbitrary and it used to be the diameter of the sphere or the equivalent diameter defined by $d = 6V/S$ for packed beds of uniform beads in other works. The plot with l_2 as the characteristic length gave a similar trend; however, it had a wider scattering than Figure 1.

To examine the validity of choosing l_1 as the characteristic length, experimental results for spherical beads by other researchers are compared in Figure 2, where Eqs. 2 and 3 are used to evaluate l_1 for spherical beads. Since the heat-transfer data for spheres are numerous (Ishimoto, 1986) showing a wide scattering, their rough envelope is drawn showing a general coincidence with our data.

Thus it is concluded that the heat transfer and the pressure drop are correlated primarily with l_1 as the characteristic length. However, slight differences among the materials in Figure 1 should be explained with other parameters such as ϵ and c.

Now we examine an equation to correlate the heat-transfer data in a general form:

$$Nu = K(1-\epsilon)^m \epsilon^n c^r (1-c)^s f(Re) \tag{7}$$

The exponents in Eq. 7 were obtained by the least-square method to yield:

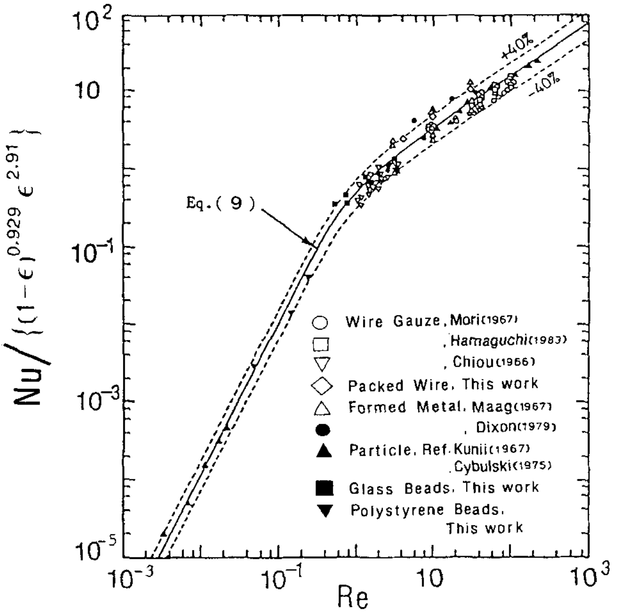

Figure 3. Comparison between the similarity correlation (Eq. 9) with the experimental data.

$$Nu = 0.430(1-\epsilon)^{0.929}\epsilon^{2.91}c^{0.0795}Re^{w},$$
$$w = 1.84 - 1.11\,Re^{0.639}/(1+Re^{0.639}). \qquad (8)$$

It is interesting to note that each exponent is very close to integer. Since the effect of c is negligible, the important parameter among those we adopted first in Eqs. 3 and 4 is only l_1, and an alternative approximate equation without the term including c is given by:

$$Nu = Nu'(1-\epsilon)^{0.929}\epsilon^{2.91}$$
$$Nu' = 0.9Re^{1.95}[1 - \exp(-0.8Re^{-1.27})]. \qquad (9)$$

Figure 3 compares our data with those of Mori (1967), Hamaguchi (1983), and Chiou (1966) for wire nets and of Cybulski (1975) and Kunii (1961) for fixed beds on Re vs. Nu' plane. The data given by Cybulski and Kunii are only for heat transfer, and the characteristic length l_l are estimated from Ergun's equation. It is shown that all of the data are correlated by Eq. 9 within ±40%.

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Notation

a = constant defined in Eq. 1
b = constant defined in Eq. 1
c = dimensionless parameter defined by Eq. 4
d = diameter
K = constant
l_i = (i = 1, 2) characteristic length defined by Eq. 3
Nu = Nusselt number defined by Eq. 6
ΔP = pressure drop
Re = Reynolds number defined by Eq. 7
S = total heat-transfer surface
V = volume of the bed

Greek letters

α_v = volumetric heat-transfer coefficient
ϵ = fractional void volume
λ = thermal conductivity
μ = viscosity
ν = kinetic viscosity
ρ = density

Subscripts

g = gas
p = particle

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